TMD Factorization for Quarkonium Production at Low Transverse Momentum

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Abstract

Quarkonium production in hadron collisions at low transverse momentum can be used for probing Transverse Momentum Dependent(TMD) gluon distributions. For this purpose one needs to establish the TMD factorization for the process. We examine the factorization at one-loop level for the production of η_c or η_b. The perturbative coefficient in the factorization is determined at one-loop accuracy. Comparing the factorization derived at tree-level and that beyond the tree-level, a soft factor is in general needed to completely cancel soft divergences. We have also discussed possible complications of TMD factorization of p-wave quarkonium production.

Quarkonium production in hadron collision can be used to explore the gluon content of hadrons, because the quarkonium is dominantly produced through gluon-gluon fusions. For the produced quarkonium with large transverse momentum, one can apply QCD collinear factorizations for long distance effects of the initial hadrons. In this case, one can extract the standard gluon distributions(see e.g., [1]). If the quakonium is produced with small transverse momentum $q_\perp$, it can be thought that the small $q_\perp$ is generated at least partly from the transverse motion of gluons inside the initial hadrons. In this case one can apply Transverse Momentum Dependent(TMD) factorization for initial hadrons. Therefore, the production with small $q_\perp$ allows to access TMD gluon distributions.

Factorizations with TMD quark distributions and fragmentation functions have been studied intensively beyond tree-level in different processes in[2, 3, 4, 5]. In comparison, the factorization with TMD gluon distributions beyond tree-level has only been studied for Higgs production in hadron collision in [6]. Recently, the TMD factorization of quarkonium production has been derived at tree-level in [7], and based on it numerical predictions have been obtained. For theoretical consistency and precision it is important to examine the TMD factorization beyond tree-level. From early studies in [2, 3, 4, 6] it is known that a soft factor needs to be implemented into the factorization. In this work we examine TMD factorization of η_c or η_b production at one-loop level.

A quarkonium is dominantly a bound state of a heavy quark Q and its antiquark $\bar{Q}$. Because of the heavy mass the $Q\bar{Q}$ pair is of a nonrelativistic system. To separate the nonperturbative effects related to the quarkonium in its production one can employ nonrelativistic QCD(NRQCD) factorization[8] by an expansion of the small velocity of Q relative to $\bar{Q}$. The inclusive production of a quarkonium at moderate- or large $q_\perp$ has been studied intensively both in theory and in experiment. In last five years, important progresses were made in the study of the next-to-leading order QCD correction for J/ψ production in hadron collisions[9] and power corrections[10]. The activities in this field can be seen in [11]. It should

be noted that in experiment it is also possible to study the inclusive production at low $q_\perp$. E.g., a J/ψ produced at LHCb can be measured with $q_\perp$ smaller than 1GeV[12]. Therefore, with theoretically established TMD factorization one can extract from experimental results TMD gluon distributions.

We will use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. We introduce two light cone vectors $n^\mu = (0, 1, 0, 0)$ and $l^\mu = (1, 0, 0, 0)$ and the transverse metric $g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu l^\nu - n^\nu l^\mu$. We consider the process:

$$h_A(P_A) + h_B(P_B) \to \eta_Q(q) + X, \tag{1}$$

in the kinematical region $Q^2 = q^2 \gg q_\perp^2$ with Q as the mass of η_Q, where η_Q stands for η_c or η_b. The momenta of the initial hadrons and of the quarkonium are given by

$$P_A^\mu \approx (P_A^+, 0, 0, 0), \quad P_B^\mu \approx (0, P_B^-, 0, 0), \quad q^\mu = (xP_A^+, yP_B^-, \vec{q}_\perp), \tag{2}$$

where we have neglected masses of hadrons, i.e., $P_A^- \approx 0$ and $P_B^+ \approx 0$. For each hadron in the initial state one can define its TMD gluon distribution. We introduce the gauge link along the direction $u^\mu = (u^+, u^-, 0, 0)$:

$$\mathcal{L}_u(z, -\infty) = P \exp\left(-ig_s \int_{-\infty}^0 d\lambda u \cdot G(\lambda u + z)\right), \tag{3}$$

where the gluon field is in the adjoint representation. At leading twist one can define two TMD gluon distributions through the gluon density matrix[13]:

$$\begin{aligned}
&\frac{1}{xP^+} \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ix\xi^- P_A^+ + i\vec{\xi}_\perp \cdot \vec{k}_\perp} \langle h_A | \left(G^{+\mu}(\xi)\mathcal{L}_u(\xi, -\infty)\right)^a \left(\mathcal{L}_u^\dagger(0, -\infty) G^{+\nu}(0)\right)^a | h_A \rangle \\
&= -\frac{1}{2} g_\perp^{\mu\nu} f_{g/A}(x, k_\perp, \zeta_u^2, \mu) + \left(k_\perp^\mu k_\perp^\nu + \frac{1}{2} g_\perp^{\mu\nu} k_\perp^2\right) h_{g/A}(x, k_\perp, \zeta_u^2, \mu)
\end{aligned} \tag{4}$$

with $\xi^\mu = (0, \xi^-, \vec{\xi}_\perp)$. x is the momentum fraction carried by the gluon inside h_A. The gluon has also a nonzero transverse momentum $\vec{k}_\perp$. The definition is given in non-singular gauges. It is gauge invariant. In singular gauges, one needs to add gauge links along transverse direction at $\xi^- = -\infty$[14]. Because of the gauge links, the TMD gluon distributions also depend on the vector u through the variable:

$$\zeta_u^2 = \frac{(2u \cdot P_A)^2}{u^2} \approx \frac{2u^-}{u^+} \left(P_A^+\right)^2. \tag{5}$$

In the definition the limit $u^+ \ll u^-$ is taken in the sense that one neglects all contributions suppressed by negative powers of ζ_u^2.

From the definition in Eq.(4) there are two TMD gluon distributions. The distribution $f_{g/A}$ corresponds to the standard gluon distribution in collinear factorization. The distribution $h_{g/A}$ describes gluons with linear polarization inside h_A. The relevant phenomenology of $h_{g/A}$ has been only recently studied[15, 16]. Through the process studied here one can also obtain information about this distribution [7]. For h_B one can also define two TMD gluon distributions $f_{g/B}$ and $h_{g/B}$ similar to those in Eq.(4), in which the gauge links are along the direction $v^\mu = (v^+, v^-, 0, 0)$ instead of u and the limit $v^+ \gg v^-$ is taken. Therefore the two distributions $f_{g/B}$ and $h_{g/B}$ depend on the parameter ζ_v which is defined by replacing in ζ_u P_A with P_B and u with v in Eq.(5).

To study the TMD factorization of the process in Eq.(1), we need to study

$$g(p) + g(\bar{p}) \to \eta_Q(q) + X, \tag{6}$$

with $p^\mu = (P_A^+, 0, 0, 0)$ and $\bar{p}^\mu = (0, P_B^-, 0, 0)$. The reason is the following: If the factorization holds, it holds for arbitrary hadrons in the initial state. Especially, it also holds if the initial states are of partons. Correspondingly, we need to study the TMD gluon distributions of the corresponding gluon. That is, we need to study $f_{g/A}$ and $h_{g/A}$ of the gluon $g(p)$ instead of h_A and $f_{g/B}$ and $h_{g/B}$ of the gluon $g(\bar{p})$ instead of h_B . At tree-level, one easily finds:

$$f^{(0)}_{g/A}(x, k_\perp, \zeta_u^2, \mu) = f^{(0)}_{g/B}(x, k_\perp, \zeta_v^2, \mu) = \delta(1-x)\delta^2(\vec{k}_\perp), \tag{7}$$

while $h_{g/A}$ and $h_{g/B}$ are zero. They become nonzero at order of α_s.

In this work we will work at the leading order of the small velocity expansion in NRQCD. At this order the production of η_Q can be thought as a two-step process. In the first step a $Q\bar{Q}$ pair is produced in which the heavy quark Q and its antiquark $\bar{Q}$ carry the same momentum $q/2$. The pair is in color-singlet and spin-singlet $^1S_0$. Then the pair is transmitted into η_Q with the mass $Q = 2m_Q$. The transition is described by an NRQCD matrix element. At tree-level, the process in Eq.(6) is with X as nothing. It is straightforward to obtain the differential cross-section:

$$\begin{aligned}\frac{d\sigma}{dxdyd^2q_\perp} &= \sigma_0 \frac{\pi}{Q^2}\delta(xys - Q^2)\delta(1-x)\delta(1-y)\delta^2(\vec{q}_\perp), \\ \sigma_0 &= \frac{(4\pi\alpha_s)^2}{N_c(N_c^2-1)m_Q}|\psi(0)|^2,\end{aligned} \tag{8}$$

with $s = 2p^+\bar{p}^-$ and m_Q being the pole mass of the heavy quark. $\psi(0)$ is the wave function of η_Q at the origin. In fact $|\psi(0)|^2$ should be expressed as a NRQCD matrix element. With results in Eq.(7) one can derive the factorization:

$$\begin{aligned}\frac{d\sigma}{dxdyd^2q_\perp} &= \frac{\pi\sigma_0}{Q^2}\int d^2k_{a\perp}d^2k_{b\perp} f_{g/A}(x, k_{a\perp}) f_{g/B}(y, k_{b\perp})\delta^2(\vec{k}_{a\perp} + \vec{k}_{b\perp} - \vec{q}_\perp)\delta(xys - Q^2)\mathcal{H}, \\ \mathcal{H} &= 1 + \mathcal{O}(\alpha_s),\end{aligned} \tag{9}$$

at tree-level. Beyond the tree-level one needs to introduce a soft factor. As we will see explicitly , all soft divergences will be factorized into the soft factor and TMD gluon distributions so that the perturbative coefficient $\mathcal{H}$ is free from soft divergence. We will then determine $\mathcal{H}$ at one-loop level.

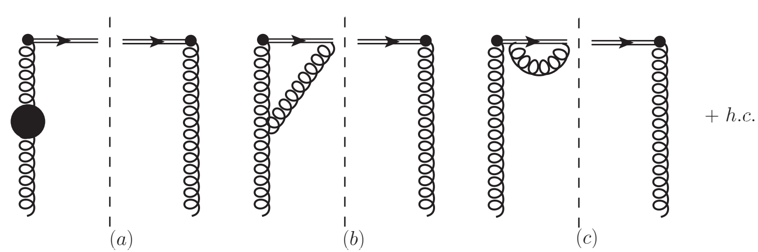

Figure 1: The one-loop corrections to the gluon TMD. The double lines represent the gauge link. The black bubble in Fig.1a is for self-energy correction.

To derive the factorization at one-loop we need to study the one-loop corrections to TMD gluon distributions and the differential cross-section. The one-loop correction to TMD gluon distribution has been studied in [6] where the collinear divergence has been regularized with a infinitely small off-shellness of the gluon. Here we regularize all divergences in $d = 4 - \epsilon$ space-time. The correction can be divided

into the virtual- and real correction. The virtual correction is given by diagrams in Fig.1. We will use $\overline{\rm MS}$-scheme to subtract U.V. divergences. After the subtraction we have the virtual correction from Fig.1:

$$
\begin{aligned}
f_{g/A}^{(1)}(x,k_\perp,\zeta_u,\mu)\Big|_{vir.} \quad = \quad & \frac{\alpha_s}{4\pi}\delta(1-x)\delta^2(\vec k_\perp)\left[\left(-\frac{2}{\epsilon_s}+\ln\frac{e^\gamma\mu^2}{4\pi\mu_s^2}\right)\left(\frac{11}{3}N_c-\frac{2}{3}N_F\right)\right.\\
& +2N_c\left(-\frac{4}{\epsilon_s^2}-\frac{2}{\epsilon_s}\ln\frac{4\pi\mu_s^2}{e^\gamma\zeta_u^2}-\frac{1}{2}\ln^2\frac{4\pi\mu_s^2}{e^\gamma\zeta_u^2}-\frac{5\pi^2}{12}+\left(-\frac{2}{\epsilon_s}+\ln\frac{e^\gamma\zeta_u^2}{4\pi\mu_s^2}\right.\right.\\
& \left.\left.\left.+\frac{1}{2}\ln\frac{\mu^2}{\zeta_u^2}-\frac{3}{2}\right)\right)\right], \qquad (10)
\end{aligned}
$$

where the poles in $\epsilon_s = 4-d$ stand for collinear- or infrared divergences, i.e., soft divergences. μ_s is the scale associated with these poles. μ is the U.V. scale. The terms in the first line in Eq.(10) is the sum of the contributions from Fig.1a and Fig.1c with their conjugated diagrams. The remaining terms are from Fig.1b and its conjugated diagram.

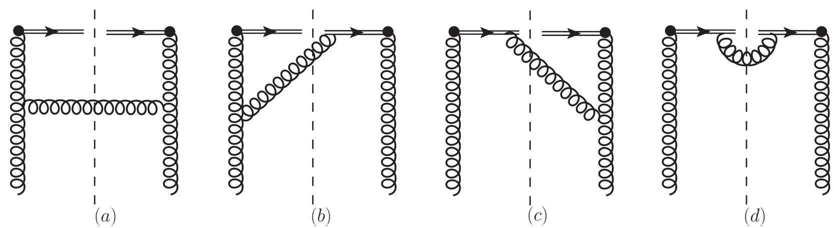

Figure 2: The real correction at one-loop to the gluon TMD. The double lines represent the gauge link. These diagrams are for real corrections.

The corrections from Fig.2 are real corrections. They can be found in [6] as

$$
\begin{aligned}
f_{g/A}^{(1)}(x,k_\perp,\zeta_u,\mu)\Big|_{re.} \quad = \quad & \frac{\alpha_s N_c}{\pi^2 k_\perp^2}\left[\left(\frac{1-x}{x}+x(1-x)+\frac{x}{2}\right)-\frac{1}{2}\delta(1-x)\right.\\
& \left.+\frac{x}{(1-x)_+}-\frac{x}{2}+\frac{1}{2}\delta(1-x)\ln\frac{\zeta_u^2}{k_\perp^2}\right], \qquad (11)
\end{aligned}
$$

where the terms in the first line are from Fig.2a and Fig.2d. The total one-loop correction is then the sum of the virtual- and real correction. At one-loop $h_{g/A}$ becomes nonzero. It receives a contribution from Fig.2a. We have:

$$
h_{g/A}(x,k_\perp,\zeta_u,\mu) = \frac{2\alpha_s N_c}{\pi^2 (k_\perp^2)^2}\frac{1-x}{x}+\mathcal{O}(\alpha_s^2). \qquad (12)
$$

By replacing ζ_u with ζ_v we obtain $f_{g/B}$ and $h_{g/B}$ from $f_{g/A}$ and $h_{g/A}$, respectively.

Now we turn to one-loop corrections of the differential cross-section. The corrections can be divided into the virtual correction and the real correction. The virtual correction is the one-loop correction to the process $g(p)+g(\bar p)\to\eta_Q(q)$. We denote the total contribution from the virtual correction as

$$
\frac{d\sigma(gg\to\eta_Q)}{dxdyd^2q_\perp}\Big|_{vir.} = \frac{1}{2s(2\pi)^3}\delta(xys-Q^2)\delta(1-x)\delta(1-y)\delta^2(\vec q_\perp)\sigma_1. \qquad (13)
$$

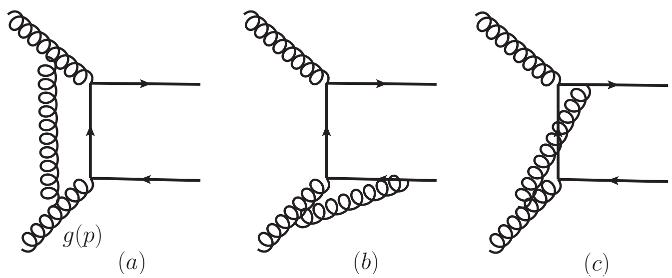

Figure 3: The class of diagrams where a gluon is emitted from the initial gluon $g(p)$ and is attached to a possible place. There are 6 diagrams. 3 of them are given here. Another 3 diagrams are obtained by reversing the direction of the heavy quark line.

The contributions to σ_1 can be divided into 4 parts:

$$\sigma_1 = \sigma_{1A} + \sigma_{1B} + \sigma_{1C} + \sigma_{1D}, \tag{14}$$

σ_{1A} receives contributions from diagrams in which a virtual gluon is emitted by the initial gluon $g(p)$. The diagrams for this part are given in Fig.3. σ_{1B} receives contributions from diagrams in which a virtual gluon is emitted by the initial gluon $g(\bar{p})$. σ_{1C} denotes the contributions from diagrams in which a virtual gluon is exchanged between heavy quark line. σ_{1D} denotes the one-loop corrections of external gluon lines. This part will not contribute to $\mathcal{H}$, because the contributions to σ_{1D} are automatically subtracted into TMD gluon distributions. In the below we will only give and discuss the results of $\sigma_{1A,1B,1C}$.

In the above classification Fig.3a can contribute both to σ_{1A} and σ_{1B}. We put the half of the contribution Fig.3a into σ_{1A} and another half into σ_{1B}. With symmetry arguments one easily finds $\sigma_{1A} = \sigma_{1B}$. We have then:

$$\sigma_{1A} = \sigma_{1B} = \frac{1}{2}\sigma_1\Big|_{3a} + \sigma_1\Big|_{3b} + \sigma_1\Big|_{3c}. \tag{15}$$

By summing contributions from each diagram we obtain the following results for the virtual corrections:

$$\begin{aligned}
\frac{\sigma_{1A}}{\sigma_0} &= \frac{\alpha_s N_c}{12\pi}\Big[-6\frac{4}{\epsilon_s^2} - 6\frac{2}{\epsilon_s}\Big(1 + \ln\frac{e^{-\gamma}4\pi\mu_s^2}{Q^2}\Big) - 3\ln^2\frac{e^{-\gamma}4\pi\mu_s^2}{Q^2} - 6\ln\frac{e^{-\gamma}4\pi\mu_s^2}{Q^2} \\
&\quad +9\ln\frac{\mu^2}{Q^2} - 6\ln 2 + 6 + \frac{11}{4}\pi^2\Big], \\
\frac{\sigma_{1C}}{\sigma_0} &= \frac{\alpha_s}{2\pi}\Big[-N_c\ln\frac{\mu^2}{Q^2} + C_F\Big(-2 + 4\ln 2\Big) + \frac{1}{N_c}\Big(2\ln 2 - \frac{1}{4}\pi^2\Big)\Big].
\end{aligned} \tag{16}$$

In these results the U.V. poles are subtracted in the $\overline{\rm MS}$-scheme. The on-shell scheme for the renormalization of heavy quark propagators is used so that $m_Q = Q/2$ is the pole mass of heavy quark. In σ_{1A} the pole terms of ϵ_s are for soft divergences coming only from Fig.3a. The contributions from Fig.3b and Fig.3c also contain collinear divergences and infrared divergences. The infrared divergences are canceled in the sum of the two diagrams, because the $Q\bar{Q}$ is in color singlet. The collinear divergences are also canceled. In calculating the diagrams for σ_{1C} one will meet Coulomb singularity. This singularity is factorized into NRQCD matrix element. Hence we have finite σ_{1C}.

The real correction is from the tree-level process

$$g(p) + g(\bar{p}) \to \eta_Q(q) + g(k). \tag{17}$$

For the color-single $Q\bar{Q}$-pair there are 12 diagrams for the amplitude. Since we are interested in the low $q_\perp$ region, we expand the differential cross section in $q_\perp/Q$ and only take the leading order in the expansion. At the leading order we have only those diagrams given in Fig.4 for the differential cross-section. The result for the process in Eq.(17) in the limit of $q_\perp \to 0$ is:

$$\begin{aligned}\frac{d\sigma}{dxdyd^2q_\perp} &= \frac{\pi\sigma_0}{Q^2}\frac{N_c\alpha_s}{4\pi^2 q_\perp^2}\delta(xys - Q^2)\left[\frac{2\delta(1-y)}{x}\left(2 - 2x + 3x^2 - 2x^3\right) + \frac{x(1+x)}{(1-x)_+}\delta(1-y)\right.\\ &\quad \left. -\delta(1-x)\delta(1-y)\ln\frac{q_\perp^2}{Q^2} + (x \leftrightarrow y)\right] + \mathcal{O}(q_\perp^0).\end{aligned} \tag{18}$$

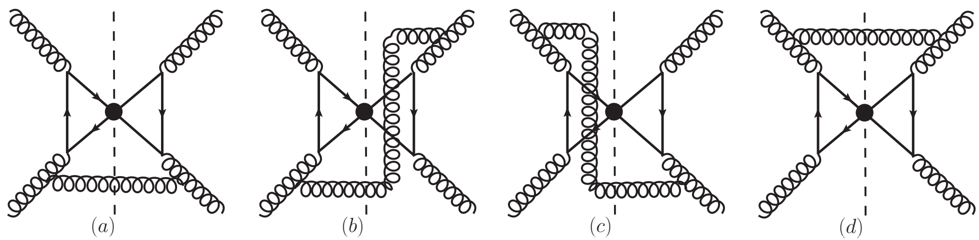

Figure 4: The diagrams for the cross-section of $g + g \to g + \eta_Q$. In these diagrams, the gluon in the intermediate state is emitted or absorbted by gluons. The black dots denote the projection of the $Q\bar{Q}$ pair into the color singlet ${}^1S_0$ state. By resversing the quark lines one can obtain other 3 diagrams from each diagram.

The factorized result in Eq.(9) is derived at tree-level. If we extent the factorization beyond tree-level, with the one-loop results in the above we will find the following: 1). The soft divergences are not factorized, i.e., $\mathcal{H}$ will contain some infrared divergences represented by poles in ϵ_s. 2). The real correction of the differential cross-section is not totally generated by TMD gluon distributions, in other word, $\mathcal{H}$ will receives correction from the real correction. It results in that $\mathcal{H}$ depends on $q_\perp$. All of these have the common reason. In the one-loop corrections to the differential cross-section there are exchange of a soft gluon between the two initial gluons $g(p)$ and $g(\bar{p})$. In the virtual correction the exchange results in infrared divergences, and in the real correction it results in contributions proportional to $\delta(1-x)\delta(1-y)$. The effects of the soft gluon exchange are not exactly generated by the corresponding soft gluon exchange in TMD gluon distributions. The effects of soft gluon exchange are of long-distance. Therefore, one needs to introduce a soft factor in the factorization to completely factorize these effects from $\mathcal{H}$ determined with Eq.(9).

The effects of soft gluon exchange between a gluon moving in the $+$-direction and a gluon moving in the $-$-direction can be described by the expectation value of a product with four gauge links. We introduce as in [6]:

$$S(\vec{b}_\perp, \mu, \rho) = \frac{1}{N_c^2 - 1}\langle 0|\mathrm{Tr}\left[\mathcal{L}_v^\dagger(\vec{b}_\perp, -\infty)\mathcal{L}_u(\vec{b}_\perp, -\infty)\mathcal{L}_u^\dagger(\vec{0}, -\infty)\mathcal{L}_v(\vec{0}, -\infty)\right]|0\rangle. \tag{19}$$

The gauge links are past-pointing. It reflects the fact that the two gluons $g(p)$ and $g(\bar{p})$ are in the initial state. The dependence on the directions of gauge links is only through the parameter $\rho^2 =$

$(2u\cdot v)^2/(u^2v^2) \approx u^-v^+/(u^+v^-)$. The limit $u^- \ll u^+$ and $v^+ \gg v^-$ is taken similarly to that in TMD gluon distributions. The gauge links or the gauge field is in the adjoint representation. At leading order one has

$$S^{(0)}(\vec{b}_\perp, \mu, \rho) = 1. \tag{20}$$

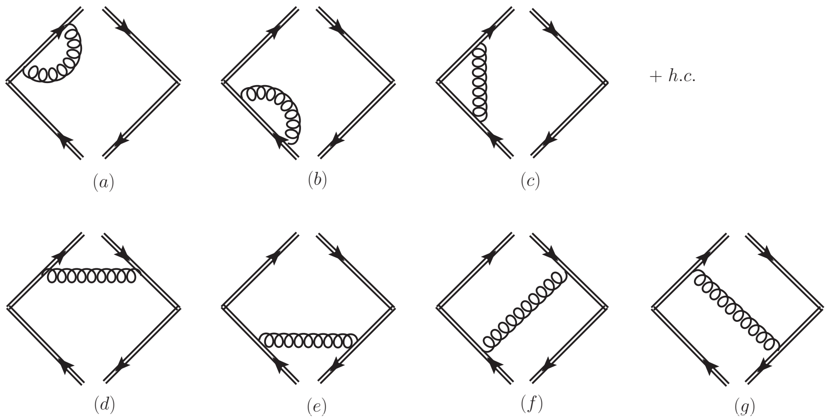

Figure 5: One-loop corrections for the soft factor. The first three diagrams plus their complex conjugated are virtual corrections. The last four diagrams are real corrections. A cut line is implied.

At one-loop there are corrections from Fig.5. One can divide the corrections into a virtual- and a real part. The diagrams in the first row are of the virtual part. Those in the second row are of the real part. The virtual correction read:

$$S^{(1)}_{vir.}(\vec{b}_\perp, \mu, \rho) = \frac{\alpha_s N_c}{2\pi}\left[-\frac{2}{\epsilon_s} + \ln\frac{e^\gamma \mu^2}{4\pi\mu_s^2}\right]\left(2 - \ln\rho^2\right), \tag{21}$$

where the U.V. pole is subtracted. The pole in ϵ_s represents the I.R. divergence with the scale μ_s. The real part is:

$$S^{(1)}_{re.}(\vec{b}_\perp, \mu, \rho) = -\frac{\alpha_s N_c}{2\pi^2}\left(2 - \ln\rho^2\right)\int d^2k_\perp \frac{e^{-i\vec{b}_\perp\cdot\vec{k}_\perp}}{k_\perp^2}. \tag{22}$$

The total one-loop contribution is the sum of the virtual- and real contribution.

We now define our soft factor which will enter the TMD factorization as:

$$\begin{aligned}\tilde{S}(\vec{\ell}_\perp, \mu, \rho) &= \int \frac{d^2b_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\ell}_\perp} S^{-1}(\vec{b}_\perp, \mu, \rho) \\ &= \delta^2(\vec{\ell}_\perp) - \frac{\alpha_s N_c}{2\pi}\left(2 - \ln\rho^2\right)\left[\left(-\frac{2}{\epsilon_s} + \ln\frac{e^\gamma\mu^2}{4\pi\mu_s^2}\right)\delta^2(\vec{\ell}_\perp) - \frac{1}{\pi\ell_\perp^2}\right] + \mathcal{O}(\alpha_s^2).\end{aligned} \tag{23}$$

With the introduced soft factor we propose the TMD factorization as:

$$\begin{aligned}\frac{d\sigma}{dxdyd^2q_\perp} &= \frac{\pi\sigma_0}{Q^2}\int d^2k_{a\perp}d^2k_{b\perp}d^2\vec{\ell}_\perp \delta^2(\vec{k}_{a\perp} + \vec{k}_{b\perp} + \vec{\ell}_\perp - \vec{q}_\perp)\delta(xys - Q^2) \\ &\quad \cdot f_{g/A}(x, k_{a\perp}, \zeta_u, \mu) f_{g/B}(y, k_{b\perp}, \zeta_v, \mu)\tilde{S}(\ell_\perp, \mu, \rho)\mathcal{H}(Q, \mu, \zeta_u, \zeta_v).\end{aligned} \tag{24}$$

From one-loop results of the differential cross-section, TMD gluon distributions and the soft factor we derive:

$$\mathcal{H}(Q,\mu,\zeta_u,\zeta_v) = 1+\frac{\alpha_s N_c}{4\pi}\left[\ln^2\frac{\zeta_u^2}{Q^2}+\ln^2\frac{\zeta_v^2}{Q^2}-\ln\rho^2\left(1+2\ln\frac{\mu^2}{Q^2}\right)+2\ln\frac{\mu^2}{Q^2}+\frac{7}{2}\pi^2 +\frac{2}{N_c^2}\left(1-\frac{1}{4}\pi^2\right)\right]+\mathcal{O}(\alpha_s^2). \quad (25)$$

It is clear that $\mathcal{H}$ is free from any soft-divergence and does not depend on $q_\perp$. With the factorization the small transverse momentum $q_\perp$ is generated by the transverse motion of gluons in the initial hadrons and by soft gluon radiation. Eq.(24) and Eq.(25) are of our main results. It should be noted that the factorization holds for arbitrary large ζ_u and ζ_v. For practical applications one may take a frame to simplify the results in Eq.(24) and Eq.(25). One can take $\zeta_u^2=\zeta_v^2=\rho Q^2$ so that the TMD gluon distributions in Eq.(24) depends on ρ and Q^2 and the perturbative coefficient becomes a function of Q, μ and ρ:

$$\mathcal{H}(Q,\mu,\rho)=1+\frac{\alpha_s N_c}{2\pi}\left[\ln^2\rho-\ln\rho\left(1+2\ln\frac{\mu^2}{Q^2}\right)+\ln\frac{\mu^2}{Q^2}+\frac{7}{4}\pi^2+\frac{1}{N_c^2}\left(1-\frac{1}{4}\pi^2\right)\right]+\mathcal{O}(\alpha_s^2). \quad (26)$$

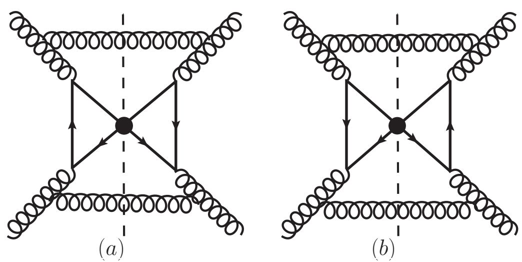

Figure 6: The diagrams for the cross-section of $g+g\to g+g+\eta_Q$ give the contributions factorized with the gluon TMD g_k. By resversing the quark lines one can obtain other 3 diagrams from each diagram.

At the orders we have consider we do not find the contributions which are factorized with $h_{g/A}$ or $h_{g/B}$ defined in Eq.(4). But, there is a contribution involving these distributions of linearly polarized gluons in the TMD factorization. This contribution can be found at higher order of α_s from diagrams given in Fig.6. It is straightforward to calculate these diagrams in the limit $q_\perp\to 0$. We find that the contribution takes the factorized form:

$$\left.\frac{d\sigma}{dxdyd^2q_\perp}\right|_{Fig.6} = \frac{\pi\sigma_0}{Q^2}\delta(xys-Q^2)\int d^2k_{a\perp}d^2k_{b\perp}\delta^2(\vec{k}_{a\perp}+\vec{k}_{b\perp}-\vec{q}_\perp)\left[f_{g/A}(x,k_{a\perp})\Big|_{2a} f_{g/B}(y,k_{b\perp})\Big|_{2a} -\frac{1}{2}\left((\vec{k}_{a\perp}\cdot\vec{k}_{b\perp})^2-\frac{1}{2}k_{a\perp}^2k_{b\perp}^2\right)h_{g/A}(x,k_{a\perp})h_{g/B}(y,k_{b\perp})\right]. \quad (27)$$

Our result in the last line has also been derived in [7] with a different method. The perturbative coefficient of the contribution in the last line is at order of α_s^0. This contribution should be added to Eq.(24). In principle one can determine the perturbative coefficient of the contribution beyond the leading order of α_s following the same way as has been done for Eq.(24,25). But this will be very tedious because one needs to calculated the partonic process $g+g\to\eta_Q+X$ at 3-loop level. We leave this for future study.

The studied TMD factorization can be used for the region with $q_\perp \sim \Lambda_{QCD}$ for extracting TMD gluon distributions. But its usage is not limited to this kinematic region, because the factorization holds in general in the region $q_\perp/Q \ll 1$. In the region $Q \gg q_\perp \gg \lambda_{QCD}$ both TMD factorization and collinear factorization hold. In the collinear factorization the perturbative coefficient functions in this region contain large log of $q_\perp/Q$. The results from TMD factorization can be used to resume these large log's. This leads to the well-known CSS resumation[3]. Based on our result here one can also derive the resummation in the case of quarkonium production, similarly to that derived in [6]. We therefore do not discuss the details about the resummation here. We note that such a resummation has been studied very recently in [17]. An early work about the resummation can be found in [18], where the formation of a quarkonium from a $Q\bar{Q}$ pair is described with a color evaporation model instead of NRQCD factorization.

At the orders we have considered, the production of a p-wave quarkonium is possible. But, the TMD factorization in this case can be complicated. According to the NRQCD factorization in [8], one needs to consider not only the contribution from the production of a color singlet p-wave $Q\bar{Q}$ pair, but also the contribution of a color-octet s-wave $Q\bar{Q}$ pair. The formation of a p-wave quarkonium from the color-singlet- and the color-octet $Q\bar{Q}$ pair is at the same order in the small velocity expansion. In the case we studied here, we only need to consider the contribution from production of a color-singlet s-wave $Q\bar{Q}$ pair. At the leading power the pair decouples with soft gluons. However in the case of p-wave quarkonia, the color-singlet p-wave- and color-octet s-wave $Q\bar{Q}$ pair can emit soft gluons at leading power. To completely separate the effects of soft gluons, one may need a different soft factor than that introduced here. This is also the reason why we write our TMD factorization in Eq.(24) explicitly with the unsubtracted TMD gluon distributions and the soft factor. It is noted that one may re-define TMD gluon distributions to factorize the differential cross section only with the re-defined TMD gluon distributions[19]. Another complication with p-wave quarkonia is that one needs a gauge link for the NRQCD matrix element of the contribution from the color-octet $Q\bar{Q}$ pair to establish NRQCD factorization beyond one-loop, as shown in [20]. We will examine the TMD factorization for p-wave quarkonium in a separate publication.

To summarize: We have studied the one-loop TMD factorization of $^1S_0$-quarkonium production in hadron collision at low transverse momentum. We find that the differential cross section can be factorized with the TMD gluon distributions, the soft factor and the perturbative coefficient. The TMD gluon distributions and the soft factor are consistently defined with QCD operators, the perturbative coefficient is determined here at one-loop. In comparison with the factorization derived at tree-level, the soft factor is needed to cancel all effects of soft gluons. Our result will be useful not only for extracting TMD gluon distributions from experimental data, but also for resumming large log's of $q_\perp$ appearing in the collinear factorization.

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